Translation operations on trianguline (φ, Γ)-modules

Zichuan Wang

Indiana University

Doctoral Student Forum

ICCM 2025

January 7, 2026

Background

Notation

$E/K/\mathbb{Q}_p$ finite extensions, $\mathbf{G} := \mathrm{GL}_n$, $\mathbf{G} \supset \mathbf{B} \supset \mathbf{T}$ Borel subgroup and max'l torus, $\mathfrak{g} \supset \mathfrak{b} \supset \mathfrak{t}$ Lie algebras, $G := \mathbf{G}(K), B := \mathbf{B}(K), T := \mathbf{T}(K)$.
$K_n := K(\mu_{p^n}), K_\infty := \bigcup_{n \geqslant 1} K_n, \Gamma_K := \mathrm{Gal}(K_\infty/K), H_K := \mathrm{Gal}(\overline{K}/K_\infty)$

Background

Notation

$E/K/\mathbb{Q}_p$ finite extensions, $\mathbf{G} := \mathrm{GL}_n$, $\mathbf{G} \supset \mathbf{B} \supset \mathbf{T}$ Borel subgroup and max'l torus, $\mathfrak{g} \supset \mathfrak{b} \supset \mathfrak{t}$ Lie algebras, $G := \mathbf{G}(K), B := \mathbf{B}(K), T := \mathbf{T}(K)$.
$K_n := K(\mu_{p^n}), K_\infty := \bigcup_{n\geqslant 1} K_n, \Gamma_K := \mathrm{Gal}(K_\infty/K), H_K := \mathrm{Gal}(\overline{K}/K_\infty)$

p-adic Langlands correspondence

$$\left\{\begin{matrix} n\text{-dim'l continuous } E\text{-linear} \\ \text{representations of } \mathrm{Gal}_K \end{matrix}\right\} \overset{?}{\longleftrightarrow} \left\{\begin{matrix} \text{some unitary } E\text{-linear Banach} \\ \text{representations of } \mathrm{GL}_n(K) \end{matrix}\right\}$$

Background

Notation

$K_n := K(\mu_{p^n}), K_\infty := \bigcup_{n\geqslant 1} K_n, \Gamma_K := \mathrm{Gal}(K_\infty/K), H_K := \mathrm{Gal}(\overline{K}/K_\infty)$
$\mathcal{R}_K :=$ analytic functions on closed annuli of closed unit disc over $K_{\infty,0}$.

Locally analytic p-adic Langlands correspondence

$$\begin{array}{ccc}
\left\{\begin{array}{c}n\text{-dim'l continuous } E\text{-linear}\\ \text{representations of } \mathrm{Gal}_K\end{array}\right\} & \overset{?}{\longleftrightarrow} & \left\{\begin{array}{c}\text{some unitary } E\text{-linear Banach}\\ \text{representations of } \mathrm{GL}_n(K)\end{array}\right\} \\
\Big\downarrow & & \Big\downarrow \text{l.a.vectors} \\
\left\{\begin{array}{c}\text{rank } n\ (\varphi,\Gamma_K)\text{-}\\ \text{modules over } \mathcal{R}_{K,E}\end{array}\right\} & \overset{?}{\longleftrightarrow} & \left\{\begin{array}{c}\text{some locally } \mathbb{Q}_p\text{-analytic}\\ \text{representations of } \mathrm{GL}_n(K)\end{array}\right\}
\end{array}$$

Background

Notation

$K_n := K(\mu_{p^n}), K_\infty := \bigcup_{n\geqslant 1} K_n, \Gamma_K := \mathrm{Gal}(K_\infty/K), H_K := \mathrm{Gal}(\overline{K}/K_\infty)$
$\mathcal{R}_K :=$ analytic functions on closed annuli of closed unit disc over $K_{\infty,0}$.
$X_{\hat{K}^\flat_\infty} := \big(\mathrm{Spa}(W(\mathcal{O}_{\hat{K}^\flat_\infty}))\backslash\{p[p^\flat]=0\}\big)/\varphi^{\mathbb{Z}}$, $[\infty]=$ the untilt $\hat{K}_\infty$ of $\hat{K}^\flat_\infty$

Locally analytic p-adic Langlands correspondence

$$\left\{\begin{matrix}\text{rank } n\ (\varphi,\Gamma_K)\text{-}\\ \text{modules over } \mathcal{R}_{K,E}\end{matrix}\right\} \xleftrightarrow[?]{D\leftrightarrow\pi(D)} \left\{\begin{matrix}\underline{\text{some}}\text{ locally } \mathbb{Q}_p\text{-analytic}\\ \text{representations of } \mathrm{GL}_n(K)\end{matrix}\right\}$$

expected to satisfy **compatibility** between Hodge-Tate(-Sen) weights and infinitesimal characters via $\mathcal{Z}(\mathfrak{g}) \xrightarrow[\simeq]{\text{Harish-Chandra}} \mathcal{O}(\mathfrak{t}^*)^W$; namely,

$$\mathbf{h}=(h_1,\dots,h_n)\mapsto \chi_{\lambda_{\mathbf{h}}},\quad \lambda_{\mathbf{h}} := \mathbf{h}-\rho.$$

Sen theory on Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$

$$\begin{array}{ccc}
\text{(φ,Γ_K)-module} & & \text{Γ_K-equivariant vector bundle} \\
D \text{ over } \mathcal{R}_K & \longleftrightarrow & \mathcal{E} \text{ on } X_{\hat{K}_\infty^\flat} \\
\Big\downarrow \text{localization} & & \Big\downarrow \text{localize at } \infty \\
\text{Γ_K-representation} & & \text{completed stalk at } \infty \circlearrowleft \Gamma_K \\
D_{\mathrm{dif}}^+ \text{ over } K_\infty[[t]] & \longleftrightarrow & \hat{\mathcal{E}}_\infty \text{ over } \hat{\mathcal{O}}_{X,\infty}
\end{array}$$

Sen theory on Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$

$$\begin{array}{ccc}
\text{(φ, Γ_K)-module} & & \text{Γ_K-equivariant vector bundle} \\
D \text{ over } \mathcal{R}_K & \longleftrightarrow & \mathcal{E} \text{ on } X_{\hat{K}_\infty^\flat} \\
\Big\downarrow \text{localization} & & \Big\downarrow \text{localize at } \infty \\
\text{Γ_K-representation} & & \text{completed stalk at } \infty \circlearrowleft \Gamma_K \\
D_{\mathrm{dif}}^+ \text{ over } K_\infty[[t]] & \longleftrightarrow & \hat{\mathcal{E}}_\infty \text{ over } \hat{\mathcal{O}}_{X,\infty} \\
\Big\downarrow \text{mod } t & & \Big\downarrow \text{fiber} \\
\mathrm{Lie}(\Gamma_K) \ni \Theta\text{-module} & & \text{fiber at } \infty \circlearrowleft \Gamma_K \\
D_{\mathrm{Sen}} := D_{\mathrm{dif}}^+/t \text{ over } K_\infty & \longleftrightarrow & \mathcal{E}|_{\{\infty\}} \text{ over } k(\infty)
\end{array}$$

Sen theory on Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$

$$
\begin{array}{ccc}
\text{(φ, Γ_K)-module } D \text{ over } \mathcal{R}_K & \longleftrightarrow & \text{Γ_K-equivariant vector bundle } \mathcal{E} \text{ on } X_{\hat{K}_\infty^\flat} \\
\big\downarrow \text{ localization} & & \big\downarrow \text{ localize at } \infty \\
\text{Γ_K-representation } D_{\mathrm{dif}}^+ \text{ over } K_\infty[[t]] & \longleftrightarrow & \text{completed stalk at } \infty \circlearrowleft \Gamma_K \ \ \hat{\mathcal{E}}_\infty \text{ over } \hat{\mathcal{O}}_{X,\infty} \\
\big\downarrow \text{ mod } t & & \big\downarrow \text{ fiber} \\
\mathrm{Lie}(\Gamma_K) \ni \Theta\text{-module } D_{\mathrm{Sen}} := D_{\mathrm{dif}}^+/t \text{ over } K_\infty & \longleftrightarrow & \text{fiber at } \infty \circlearrowleft \Gamma_K \ \ \mathcal{E}|_{\{\infty\}} \text{ over } k(\infty)
\end{array}
$$

$\rightsquigarrow$ Sen polynomial $P_{\mathrm{Sen}}(T) := \mathrm{char.poly}(\Theta) \in K[T]$

$\rightsquigarrow$ (Hodge-Tate-)Sen weights $\mathrm{wt}(D) := \mathrm{zeros}(P_{\mathrm{Sen}}(T))$.

Beauville-Laszlo and Change of weights

Consider the restrictions

Γ_K-equivariant bundle
$\mathcal{E}$ on $X_{\hat{K}^\flat_\infty}$

↙ ↘

completed stalk at ∞
$\hat{\mathcal{E}}_\infty$ over $\hat{\mathcal{O}}_{X,\infty}$

Γ_K-equivariant bundle
$\mathcal{E}|_{X\setminus\{\infty\}}$ on $X_{\hat{K}^\flat_\infty}\setminus\{\infty\}$

Beauville-Laszlo and Change of weights

Consider the restrictions

Γ_K-equivariant bundle
$\mathcal{E}$ on $X_{\hat{K}^\flat_\infty}$

↙ ↘

completed stalk at ∞
$\hat{\mathcal{E}}_\infty$ over $\hat{\mathcal{O}}_{X,\infty}$

Γ_K-equivariant bundle
$\mathcal{E}|_{X\setminus\{\infty\}}$ on $X_{\hat{K}^\flat_\infty}\setminus\{\infty\}$

Beauville-Laszlo $\rightsquigarrow D \longmapsto (D^+_{\mathrm{dif}}, D[1/t])$ defines an equivalence.
"Change of weights" $=$ Γ_K-equivariant modification at ∞ of the lattice D^+_{dif} inside $D[1/t] \otimes_{\mathcal{R}[1/t]} K_\infty((t))$.

Beauville-Laszlo and Change of weights

Consider the restrictions

Γ_K-equivariant bundle
$\mathcal{E}$ on $X_{\hat{K}^\flat_\infty}$

↙ ↘

completed stalk at ∞
$\hat{\mathcal{E}}_\infty$ over $\hat{\mathcal{O}}_{X,\infty}$

Γ_K-equivariant bundle
$\mathcal{E}|_{X\setminus\{\infty\}}$ on $X_{\hat{K}^\flat_\infty}\setminus\{\infty\}$

Beauville-Laszlo $\rightsquigarrow D \longmapsto (D^+_{\mathrm{dif}}, D[1/t])$ defines an equivalence.
"Change of weights" $=\!=$ Γ_K-equivariant modification at ∞ of the lattice D^+_{dif} inside $D[1/t] \otimes_{\mathcal{R}[1/t]} K_\infty((t))$.

Work with coefficients in affinoid E-algebras A.

Change of weights for trianguline (φ, Γ)-modules

If D comes with a B-structure $0 \subsetneq \mathrm{Fil}^1(D) \subsetneq \cdots \subsetneq \mathrm{Fil}^n(D) = D$ with graded pieces $\mathrm{Fil}^i(D)/\mathrm{Fil}^{i-1}(D) \cong \mathcal{R}(\delta_i)$ for continuous $\delta_i : K^\times \to A^\times$, then $\mathrm{wt(D)}$ are given by derivatives $\mathrm{wt(\delta_i)} = \frac{\mathrm{d}}{\mathrm{d}\gamma}(\delta_\mathrm{i})\Big|_{\gamma=1}$ of the δ_i.

Change of weights for trianguline (φ, Γ)-modules

If D comes with a B-structure $0 \subsetneq \mathrm{Fil}^1(D) \subsetneq \cdots \subsetneq \mathrm{Fil}^n(D) = D$ with graded pieces $\mathrm{Fil}^i(D)/\mathrm{Fil}^{i-1}(D) \cong \mathcal{R}(\delta_i)$ for continuous $\delta_i : K^\times \to A^\times$, then $\mathrm{wt(D)}$ are given by derivatives $\mathrm{wt}(\delta_\mathrm{i}) = \frac{\mathrm{d}}{\mathrm{d}\gamma}(\delta_\mathrm{i})\Big|_{\gamma=1}$ of the δ_i.

We may view $D \in \mathrm{Ext}^1_{\varphi,\Gamma}(Q_i, \mathrm{Fil}^i(D))$ and pullback along $t^k Q_i \subset Q_i$, where Q_i denotes the quotient (φ, Γ)-module $D/\mathrm{Fil}^i(D)$, e.g., $i = 2$

$$\begin{array}{ccccccccc} 0 \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow 0 & & (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3), \mathrm{wt}(\delta_4)) \\ & \Vert & & \rightsquigarrow & & \uparrow & & & \downarrow \\ 0 \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p_k(D) & \longrightarrow & t^k(\delta_3 - \delta_4) & \longrightarrow 0 & & (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3)+k, \mathrm{wt}(\delta_4)+k) \end{array}$$

Change of weights for trianguline (φ, Γ)-modules

If D comes with a B-structure $0 \subsetneq \mathrm{Fil}^1(D) \subsetneq \cdots \subsetneq \mathrm{Fil}^n(D) = D$ with graded pieces $\mathrm{Fil}^i(D)/\mathrm{Fil}^{i-1}(D) \cong \mathcal{R}(\delta_i)$ for continuous $\delta_i : K^\times \to A^\times$, then $\mathrm{wt(D)}$ are given by derivatives $\mathrm{wt}(\delta_\mathrm{i}) = \frac{\mathrm{d}}{\mathrm{d}\gamma}(\delta_\mathrm{i})\Big|_{\gamma=1}$ of the δ_i.

We may view $D \in \mathrm{Ext}^1_{\varphi,\Gamma}(Q_i, \mathrm{Fil}^i(D))$ and pullback along $t^k Q_i \subset Q_i$, where Q_i denotes the quotient (φ, Γ)-module $D/\mathrm{Fil}^i(D)$, e.g., $i = 2$

$$\begin{array}{ccccccccc} 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow & 0 \\ & & \| & & \rightsquigarrow & & \uparrow & & \\ 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p_k(D) & \longrightarrow & t^k(\delta_3 - \delta_4) & \longrightarrow & 0 \end{array}$$

$$\begin{array}{c} (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3), \mathrm{wt}(\delta_4)) \\ \downarrow \\ (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3)+k, \mathrm{wt}(\delta_4)+k) \end{array}$$

- The pullback $p_k(D)$ shifts the weights of the quotient part by $k \in \mathbb{N}$.

Change of weights for trianguline (φ, Γ)-modules

If D comes with a B-structure $0 \subsetneq \mathrm{Fil}^1(D) \subsetneq \cdots \subsetneq \mathrm{Fil}^n(D) = D$ with graded pieces $\mathrm{Fil}^i(D)/\mathrm{Fil}^{i-1}(D) \cong \mathcal{R}(\delta_i)$ for continuous $\delta_i : K^\times \to A^\times$, then $\mathrm{wt(D)}$ are given by derivatives $\mathrm{wt}(\delta_\mathrm{i}) = \frac{\mathrm{d}}{\mathrm{d}\gamma}(\delta_\mathrm{i})\Big|_{\gamma=1}$ of the δ_i.

We may view $D \in \mathrm{Ext}^1_{\varphi,\Gamma}(Q_i, \mathrm{Fil}^i(D))$ and pullback along $t^k Q_i \subset Q_i$, where Q_i denotes the quotient (φ, Γ)-module $D/\mathrm{Fil}^i(D)$, e.g., $i = 2$

$$\begin{array}{ccccccccc} 0 \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow 0 & & (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3), \mathrm{wt}(\delta_4)) \\ & \Big\| & & \Big\downarrow & & \Big\uparrow & & & \Big\downarrow \\ 0 \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p_k(D) & \longrightarrow & t^k(\delta_3 - \delta_4) & \longrightarrow 0 & & (\mathrm{wt}(\delta_1), \mathrm{wt}(\delta_2), \mathrm{wt}(\delta_3)+k, \mathrm{wt}(\delta_4)+k) \end{array}$$

- The pullback $p_k(D)$ shifts the weights of the quotient part by $k \in \mathbb{N}$.
- Natural questions: (when) is it independent of the choice of B-structure, and (when) is it invertible?

Using Sen polynomials

Proposition (Zhixiang Wu, 2024)

If $P_{\mathrm{Sen},D}(T) = Q(T)S(T)$ with coprime monic $Q, S \in A[T]$, there exists unique (φ, Γ)-submodule $tD \subset D' \subset D$ with $P_{\mathrm{Sen},D'}(T) = Q(T-1)S(T)$.

Let us denote this submodule by $p(D) := D'$.

Using Sen polynomials

Proposition (Zhixiang Wu, 2024)

If $P_{\mathrm{Sen},D}(T) = Q(T)S(T)$ with coprime monic $Q, S \in A[T]$, there exists unique (φ, Γ)-submodule $tD \subset D' \subset D$ with $P_{\mathrm{Sen},D'}(T) = Q(T-1)S(T)$.

Let us denote this submodule by $p(D) := D'$. Applying this fact to $t^{-1}D$, $\rightsquigarrow \exists! D \subset q(D) \subset t^{-1}D$ of Sen polynomial $Q(T+1)S(T)$. $\rightsquigarrow q \circ p = \mathrm{id}$.

Using Sen polynomials

Proposition (Zhixiang Wu, 2024)

If $P_{\mathrm{Sen},D}(T) = Q(T)S(T)$ with coprime monic $Q, S \in A[T]$, there exists unique (φ, Γ)-submodule $tD \subset D' \subset D$ with $P_{\mathrm{Sen},D'}(T) = Q(T-1)S(T)$.

Let us denote this submodule by $p(D) := D'$. Applying this fact to $t^{-1}D$, $\rightsquigarrow \exists! D \subset q(D) \subset t^{-1}D$ of Sen polynomial $Q(T+1)S(T)$. $\rightsquigarrow q \circ p = \mathrm{id}$.

$$\begin{array}{ccccccccc}
0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow & 0 \\
 & & \| & & \wr & & \uparrow & & \\
0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p(D) & \longrightarrow & t(\delta_3 - \delta_4) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & t^{-1}(\delta_1 - \delta_2) & \longrightarrow & t^{-1}p(D) & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow & 0 \\
 & & \uparrow & & \wr & & \| & & \\
0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & \boxed{qp(D) = D} & \longrightarrow & (\delta_3 - \delta_4) & \longrightarrow & 0
\end{array}$$

Using Sen polynomials

Consequence. If we consider those trianguline (φ, Γ)-modules on $\mathrm{Sp}(A)$ that are "**weight-uniform**" at all geometric points (all B-structures there induce the same ordering of Sen weights, e.g., *non-critical crystabelline*),

Using Sen polynomials

Consequence. If we consider those trianguline (φ, Γ)-modules on $\mathrm{Sp}(A)$ that are "**weight-uniform**" at all geometric points (all B-structures there induce the same ordering of Sen weights, e.g., *non-critical crystabelline*),

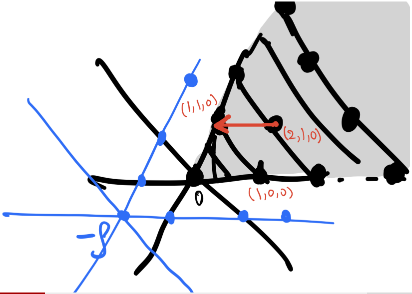
$\rightsquigarrow$ the pullback map $p_{\mathbf{k}} : \mathfrak{X}_B^{\mathrm{wu}} \to \mathfrak{X}_B^{\mathrm{wu}}$ descends to an isomorphism of *certain* substacks $p_{\mathbf{k}} : \mathfrak{X}_n^{\mathrm{wu},\mathbf{k}} \xrightarrow{\simeq} \mathfrak{X}_n^{\mathrm{wu},-\mathbf{k}}$ of the moduli stack $\mathfrak{X}_n$ of rank n Γ_K-equivariant vector bundles on the Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$.

Using Sen polynomials

Consequence. If we consider those trianguline (φ, Γ)-modules on $\mathrm{Sp}(A)$ that are "**weight-uniform**" at all geometric points (all B-structures there induce the same ordering of Sen weights, e.g., *non-critical crystabelline*),

$\rightsquigarrow$ the pullback map $p_{\mathbf{k}} : \mathfrak{X}_B^{\mathrm{wu}} \to \mathfrak{X}_B^{\mathrm{wu}}$ descends to an isomorphism of *certain* substacks $p_{\mathbf{k}} : \mathfrak{X}_n^{\mathrm{wu},\mathbf{k}} \xrightarrow{\simeq} \mathfrak{X}_n^{\mathrm{wu},-\mathbf{k}}$ of the moduli stack $\mathfrak{X}_n$ of rank n Γ_K-equivariant vector bundles on the Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$.

Without getting to the **walls**, one can move freely in the Weyl chamber.

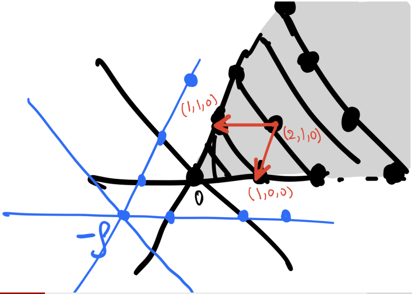


Using Sen polynomials

Consequence. If we consider those trianguline (φ, Γ)-modules on $\mathrm{Sp}(A)$ that are "**weight-uniform**" at all geometric points (all B-structures there induce the same ordering of Sen weights, e.g., *non-critical crystabelline*),

$\rightsquigarrow$ the pullback map $p_{\mathbf{k}} : \mathfrak{X}_B^{\mathrm{wu}} \to \mathfrak{X}_B^{\mathrm{wu}}$ descends to an isomorphism of *certain* substacks $p_{\mathbf{k}} : \mathfrak{X}_n^{\mathrm{wu},\mathbf{k}} \xrightarrow{\simeq} \mathfrak{X}_n^{\mathrm{wu},-\mathbf{k}}$ of the moduli stack $\mathfrak{X}_n$ of rank n Γ_K-equivariant vector bundles on the Fargues-Fontaine curve $X_{\hat{K}_\infty^\flat}$.

Without getting to the **walls**, one can move freely in the Weyl chamber.



Compatibility under Ding's crystabelline correspondence

For non-critical crystabelline (φ, Γ)-modules D over $\mathcal{R}_{K,E}$ of regular weights $\mathbf{h} = (h_1 > h_2 > \cdots > h_n) \in \mathbb{Z}^n$, with smooth characters $(\phi_i : \mathbb{Q}_p^\times \to E^\times)$ such that $D[1/t] \cong \bigoplus_{i=1}^n \mathcal{R}_{\mathbb{Q}_p,E}(\phi_i)[1/t]$, Ding has constructed a 1-1 correspondence:

Theorem (Yiwen Ding, 2024)

For $D \in \Phi\Gamma_{\mathrm{nc}}(\phi, \mathbf{h})$, there is an explicit locally analytic representation $\pi_1(D)$ of $\mathrm{GL}_n(\mathbb{Q}_p)$ that *completely determines* D, in the sense that if $D, D' \in \Phi\Gamma_{\mathrm{nc}}(\phi, \mathbf{h})$, then $D \cong D'$ precisely when $\pi_1(D) \cong \pi_1(D')$.

Compatibility under Ding's crystabelline correspondence

For non-critical crystabelline (φ, Γ)-modules D over $\mathcal{R}_{K,E}$ of regular weights $\mathbf{h} = (h_1 > h_2 > \cdots > h_n) \in \mathbb{Z}^n$, with smooth characters $(\phi_i : \mathbb{Q}_p^\times \to E^\times)$ such that $D[1/t] \cong \bigoplus_{i=1}^n \mathcal{R}_{\mathbb{Q}_p,E}(\phi_i)[1/t]$, Ding has constructed a 1-1 correspondence:

Theorem (Yiwen Ding, 2024)

For $D \in \Phi\Gamma_{\mathrm{nc}}(\phi, \mathbf{h})$, there is an explicit locally analytic representation $\pi_1(D)$ of $\mathrm{GL}_n(\mathbb{Q}_p)$ that *completely determines* D, in the sense that if $D, D' \in \Phi\Gamma_{\mathrm{nc}}(\phi, \mathbf{h})$, then $D \cong D'$ precisely when $\pi_1(D) \cong \pi_1(D')$.

From the work of Ding, Jena-Lahiri-Strauch, and Wu, it follows that

Theorem (W., 2025)

If two regular weights $\mathbf{h} \rightsquigarrow \mathbf{h}'$ are related by $p_{\mathbf{k}}$, then for $D \in \Phi\Gamma_{\mathrm{nc}}(\phi, \mathbf{h})$,

$$T_{\lambda_{\mathbf{h}}}^{\lambda_{\mathbf{h}'}}(\pi_1(D)) = \pi_1(p_{\mathbf{k}}(D)).$$

References

Yiwen Ding.
Change of weights for locally analytic representations of $\mathrm{GL}_2(\mathbb{Q}_p)$.
https://arxiv.org/abs/2307.04332.

Yiwen Ding.
p-Adic Hodge parameters in the crystabelline representations of GL_n.
Publ. Math. Inst. Hautes Études Sci., 2025.

Akash Jena, Aranya Lahiri, and Matthias Strauch.
Translation functors for locally analytic representations.
Int. J. Number Theory, 20(10):2611–2640, 2024.

Zichuan Wang.
Change of weights operations for triangulated (φ, Γ)*-modules.*
https://arxiv.org/abs/2511.15962.

Zhixiang Wu.
Geometric translations of (φ, Γ)*-modules for* $\mathrm{GL}_2(\mathbb{Q}_p)$.
https://arxiv.org/abs/2405.16637v2.

Thank you!

Yiwen Ding.
Change of weights for locally analytic representations of $\mathrm{GL}_2(\mathbb{Q}_p)$.
https://arxiv.org/abs/2307.04332.

Yiwen Ding.
p-Adic Hodge parameters in the crystabelline representations of GL_n.
Publ. Math. Inst. Hautes Études Sci., 2025.

Akash Jena, Aranya Lahiri, and Matthias Strauch.
Translation functors for locally analytic representations.
Int. J. Number Theory, 20(10):2611–2640, 2024.

Zichuan Wang.
Change of weights operations for triangulated (φ, Γ)*-modules*.
https://arxiv.org/abs/2511.15962.

Zhixiang Wu.
Geometric translations of (φ, Γ)*-modules for* $\mathrm{GL}_2(\mathbb{Q}_p)$.
https://arxiv.org/abs/2405.16637v2.