

Translation operations on trianguline (φ, Γ) -modules

Zichuan Wang

Indiana University

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Background

Notation

$E/K/\mathbb{Q}_p$ finite extensions, $\mathbf{G} := \mathrm{GL}_n$, $\mathbf{G} \supset \mathbf{B} \supset \mathbf{T}$ Borel subgroup and max'l torus, $\mathfrak{g} \supset \mathfrak{b} \supset \mathfrak{t}$ Lie algebras, $G := \mathbf{G}(K)$, $B := \mathbf{B}(K)$, $T := \mathbf{T}(K)$.
 $K_n := K(\mu_{p^n})$, $K_\infty := \bigcup_{n \geq 1} K_n$, $\Gamma_K := \mathrm{Gal}(K_\infty/K)$, $H_K := \mathrm{Gal}(\overline{K}/K_\infty)$

Locally analytic p -adic Langlands correspondence

$$\begin{array}{ccc} \left\{ \begin{array}{l} n\text{-dim'l continuous } E\text{-linear} \\ \text{representations of } \mathrm{Gal}_K \end{array} \right\} & \xleftrightarrow{?} & \left\{ \begin{array}{l} \text{some unitary } E\text{-linear Banach} \\ \text{representations of } \mathrm{GL}_n(K) \end{array} \right\} \\ \downarrow & & \downarrow \text{l.a. vectors} \\ \left\{ \begin{array}{l} \text{rank } n \text{ } (\varphi, \Gamma_K)\text{-} \\ \text{modules over } \mathcal{R}_{K,E} \end{array} \right\} & \xleftrightarrow{?} & \left\{ \begin{array}{l} \text{some locally } \mathbb{Q}_p\text{-analytic} \\ \text{representations of } \mathrm{GL}_n(K) \end{array} \right\} \end{array}$$

Background

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$K_n := K(\mu_{p^n})$, $K_\infty := \bigcup_{n \geq 1} K_n$, $\Gamma_K := \text{Gal}(K_\infty/K)$, $H_K := \text{Gal}(\overline{K}/K_\infty)$
 $\mathcal{R}_K :=$ analytic functions on closed annuli of closed unit disc over $K_{\infty,0}$.
 $X_{\hat{K}_\infty^b} := (\text{Spa}(W(\mathcal{O}_{\hat{K}_\infty^b})) \setminus \{p[p^b] = 0\}) / \varphi^{\mathbb{Z}}$, $[\infty] =$ the untilt \hat{K}_∞ of \hat{K}_∞^b

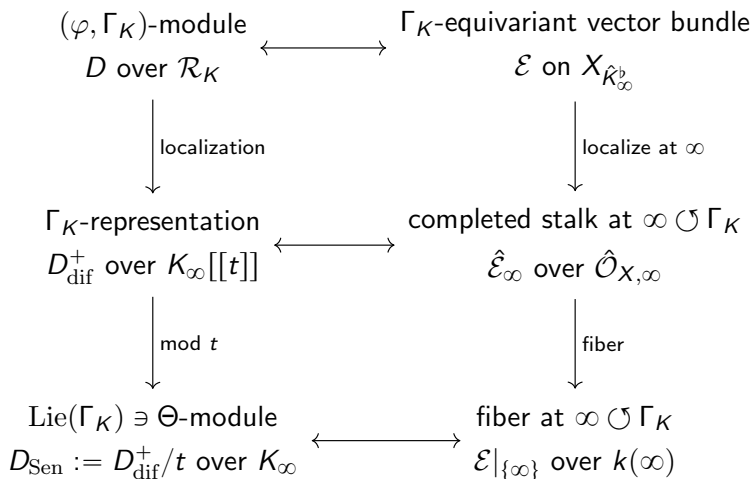
Locally analytic p -adic Langlands correspondence

$$\left\{ \begin{array}{l} \text{rank } n \text{ } (\varphi, \Gamma_K)\text{-} \\ \text{modules over } \mathcal{R}_{K,E} \end{array} \right\} \xleftrightarrow[\text{?}]{D \leftrightarrow \pi(D)} \left\{ \begin{array}{l} \text{some locally } \mathbb{Q}_p\text{-analytic} \\ \text{representations of } \text{GL}_n(K) \end{array} \right\}$$

expected to satisfy **compatibility** between Hodge-Tate(-Sen) weights and infinitesimal characters via $\mathcal{Z}(\mathfrak{g}) \xrightarrow[\simeq]{\text{Harish-Chandra}} \mathcal{O}(\mathfrak{t}^*)^W$; namely,

$$\mathbf{h} = (h_1, \dots, h_n) \mapsto \chi_{\lambda_{\mathbf{h}}}, \quad \lambda_{\mathbf{h}} := \mathbf{h} - \rho.$$

Sen theory on Fargues-Fontaine curve $X_{\hat{K}_\infty^b}$



\rightsquigarrow Sen polynomial $P_{\text{Sen}}(T) := \text{char.poly}(\Theta) \in K[T]$

\rightsquigarrow (Hodge-Tate-)Sen weights $\text{wt}(D) := \text{zeros}(P_{\text{Sen}}(T))$.

Beauville-Laszlo and Change of weights

Consider the restrictions

Γ_K -equivariant bundle

\mathcal{E} on $X_{\hat{K}_\infty^b}$

completed stalk at ∞

$\hat{\mathcal{E}}_\infty$ over $\hat{\mathcal{O}}_{X,\infty}$

Γ_K -equivariant bundle

$\mathcal{E}|_{X \setminus \{\infty\}}$ on $X_{\hat{K}_\infty^b} \setminus \{\infty\}$

Beauville-Laszlo $\rightsquigarrow D \mapsto (D_{\text{dif}}^+, D[1/t])$ defines an equivalence.

“Change of weights” = Γ_K -equivariant modification at ∞ of the lattice D_{dif}^+ inside $D[1/t] \otimes_{\mathcal{R}[1/t]} K_\infty((t))$.

Work with coefficients in affinoid E -algebras A .

Change of weights for trianguline (φ, Γ) -modules

If D comes with a B -structure $0 \subsetneq \text{Fil}^1(D) \subsetneq \cdots \subsetneq \text{Fil}^n(D) = D$ with graded pieces $\text{Fil}^i(D)/\text{Fil}^{i-1}(D) \cong \mathcal{R}(\delta_i)$ for continuous $\delta_i : K^\times \rightarrow A^\times$, then $\text{wt}(D)$ are given by derivatives $\text{wt}(\delta_i) = \left. \frac{d}{d\gamma}(\delta_i) \right|_{\gamma=1}$ of the δ_i .

We may view $D \in \text{Ext}_{\varphi, \Gamma}^1(Q_i, \text{Fil}^i(D))$ and pullback along $t^k Q_i \subset Q_i$, where Q_i denotes the quotient (φ, Γ) -module $D/\text{Fil}^i(D)$, e.g., $i = 2$

$$\begin{array}{ccccccc}
 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) \longrightarrow 0 & (\text{wt}(\delta_1), \text{wt}(\delta_2), \text{wt}(\delta_3), \text{wt}(\delta_4)) \\
 & & \parallel & & \updownarrow & & \up & \downarrow \\
 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p_k(D) & \longrightarrow & t^k(\delta_3 - \delta_4) \longrightarrow 0 & (\text{wt}(\delta_1), \text{wt}(\delta_2), \text{wt}(\delta_3)+k, \text{wt}(\delta_4)+k)
 \end{array}$$

- The pullback $p_k(D)$ shifts the weights of the quotient part by $k \in \mathbb{N}$.
- Natural questions: (when) is it independent of the choice of B -structure, and (when) is it invertible?

Using Sen polynomials

Proposition (Zhixiang Wu, 2024)

If $P_{\text{Sen},D}(T) = Q(T)S(T)$ with coprime monic $Q, S \in A[T]$, there exists unique (φ, Γ) -submodule $tD \subset D' \subset D$ with $P_{\text{Sen},D'}(T) = Q(T-1)S(T)$.

Let us denote this submodule by $p(D) := D'$. Applying this fact to $t^{-1}D$, $\rightsquigarrow \exists ! D \subset q(D) \subset t^{-1}D$ of Sen polynomial $Q(T+1)S(T)$. $\rightsquigarrow q \circ p = \text{id}$.

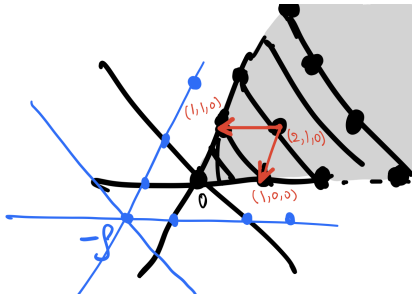
$$\begin{array}{ccccccc}
 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & D & \longrightarrow & (\delta_3 - \delta_4) \longrightarrow 0 \\
 & & \parallel & & \downarrow & & \uparrow \\
 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & p(D) & \longrightarrow & t(\delta_3 - \delta_4) \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & t^{-1}(\delta_1 - \delta_2) & \longrightarrow & t^{-1}p(D) & \longrightarrow & (\delta_3 - \delta_4) \longrightarrow 0 \\
 & & \uparrow & & \downarrow & & \parallel \\
 0 & \longrightarrow & (\delta_1 - \delta_2) & \longrightarrow & \boxed{qp(D) = D} & \longrightarrow & (\delta_3 - \delta_4) \longrightarrow 0
 \end{array}$$

Using Sen polynomials

Consequence. If we consider those trianguline (φ, Γ) -modules on $\mathrm{Sp}(A)$ that are “**weight-uniform**” at all geometric points (all B -structures there induce the same ordering of Sen weights, e.g., *non-critical crystabelline*),

\rightsquigarrow the pullback map $p_{\mathbf{k}} : \mathfrak{X}_B^{\mathrm{wu}} \rightarrow \mathfrak{X}_B^{\mathrm{wu}}$ descends to an isomorphism of *certain* substacks $p_{\mathbf{k}} : \mathfrak{X}_n^{\mathrm{wu}, \mathbf{k}} \xrightarrow{\cong} \mathfrak{X}_n^{\mathrm{wu}, -\mathbf{k}}$ of the moduli stack \mathfrak{X}_n of rank n Γ_K -equivariant vector bundles on the Fargues-Fontaine curve $X_{\hat{K}_{\infty}^b}$.

Without getting to the **walls**, one can move freely in the Weyl chamber.



Compatibility under Ding's crystabelline correspondence

For non-critical crystabelline (φ, Γ) -modules D over $\mathcal{R}_{K,E}$ of regular weights $\mathbf{h} = (h_1 > h_2 > \cdots > h_n) \in \mathbb{Z}^n$, with smooth characters $(\phi_i : \mathbb{Q}_p^\times \rightarrow E^\times)$ such that $D[1/t] \cong \bigoplus_{i=1}^n \mathcal{R}_{\mathbb{Q}_p, E}(\phi_i)[1/t]$, Ding has constructed a 1-1 correspondence:

Theorem (Yiwen Ding, 2024)

For $D \in \Phi\Gamma_{\text{nc}}(\phi, \mathbf{h})$, there is an explicit locally analytic representation $\pi_1(D)$ of $\text{GL}_n(\mathbb{Q}_p)$ that *completely determines* D , in the sense that if $D, D' \in \Phi\Gamma_{\text{nc}}(\phi, \mathbf{h})$, then $D \cong D'$ precisely when $\pi_1(D) \cong \pi_1(D')$.

From the work of Ding, Jena-Lahiri-Strauch, and Wu, it follows that

Theorem (W., 2025)

If two regular weights $\mathbf{h} \rightsquigarrow \mathbf{h}'$ are related by $\rho_{\mathbf{k}}$, then for $D \in \Phi\Gamma_{\text{nc}}(\phi, \mathbf{h})$,

$$T_{\lambda_{\mathbf{h}}}^{\lambda_{\mathbf{h}'}}(\pi_1(D)) = \pi_1(\rho_{\mathbf{k}}(D)).$$

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